Haptic stimulus for the discrimination between intrinsic properties of dynamic systems

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Abstract. This paper presents a haptic stimulus that enables the discrimination between linear and non-linear functions. These functions correspond to the evolution of divergency for two important states of dynamic systems: quasi-periodic and chaotic states respectively. The proposed approach combines a kinaesthetic feedback, to attract the user on explored attractors, and a vibrotactile feedback, to effectively display the local divergency of the attractor. The experimental results show an improvement of the level of discrimination between the two states and a quicker understanding of the right nature of the signal.

Keywords: Dynamical System Theory, Ambiguity, Haptic

1 Introduction

The study of dynamic systems presents a real challenge since standard analysis approaches (e.g., representation with attractors) provide non-trivial representations for human users. Thus, elementary analysis tasks, such as the discrimination between the several system fundamental properties, become very complex. For instance, even for expert users, the difference between the representations of quasi-periodic and chaotic attractors is sometimes hidden in details. Only a deep and uncertain analysis can reveal these differences, and the help of external informations may be needed. Nevertheless, this kind of properties is fundamental, as predictability will derive from the quasi-periodicity, and on the other hand, chaos means unpredictability.

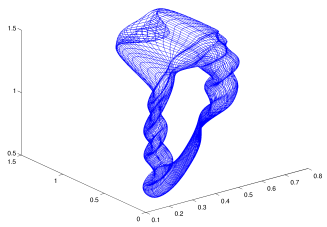
The proposed approach to improve the discrimination between the different states of dynamic systems (e.g., quasi-periodicity state, chaos state) consists in combining the 3D visual representations of attractors, or any other kind of relevant physical information, with a suitable haptic feedback leading to complementary information. The use of haptic feedback to display the local divergency of attractors is an appropriate solution since it gives access to a local feature, which may effectively supplement the visual display of spatial representation of attractors.

One can see the divergency as the evolution, over time, of two initially close trajectories. This factor involves two important variables. The first one is no more than the euclidean distance between the current states of the trajectories, while the second one is the velocity of this distance's evolution. Based on these variables, the theoretical expressions of quasi-periodicity and chaos [3, 5], two important states of dynamical systems, can be reduced as follows (see Figure 1):

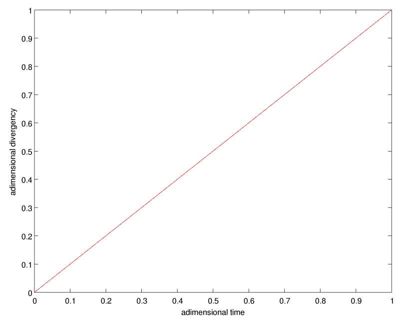
- $f(t) \propto t$: express a linear divergency rate ($\propto$ means proportional).
- $f(t) \propto \exp(t)$: express a non-linear (i.e., exponential) divergency rate.

In the rate of divergency is hidden the expression of the famous Lyapunov exponent [3–5]. It is to note that a linear divergency rate does not imply a linear system. The linear rate exposes that the future of a state is predictable, even with uncertainties both on measures and parameters of the system.

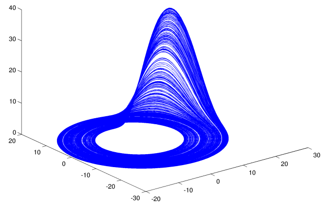
The objective of this work is to design a haptic function that enables an efficient discrimination among the two states presented above.



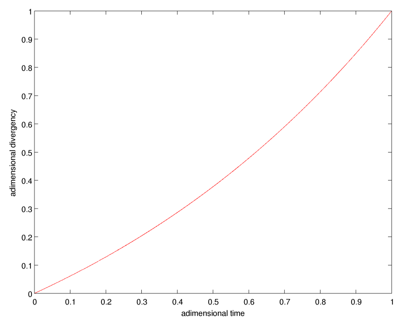
Quasi-periodic attractor



divergency over time corresponding to a quasi-periodic attractor



Chaotic attractor



divergency over time corresponding to a chaotic attractor

Fig. 1. Attractors and corresponding divergency over Time

Even if the difference between the two signals may look obvious, at *short* times or for slow growth rate, the expression of the chaos is actually reduced to the expression of the quasi-periodicity. The first-order Taylor series of the exponential function highlights this phenomenon. Accordingly, previous works showed kinaesthetic feedback presents some limitations for the discrimination between these kinds of signals [6]. To improve the level of discrimination, we propose to apply an other signal, $\hat{f}$, derived from f. $\hat{f}$ is defined as

$$\hat{f}(t) \propto \underbrace{f(t)}_{Exploration} + \underbrace{\cos\left(\frac{\mathrm{d}f}{\mathrm{d}t}(t)\,t\right)}_{Discrimination}$$

The first part of the signal (i.e., $f(t)$) corresponds to both the haptic attraction feedback required for the exploration procedure [2, 1] and the divergency expression. Actually, the exploration of physical state space must remain the core part of our approach. This attraction function is based on the linear and non-linear force models according to the attractor's type. This haptic feedback gives a first information for the discrimination between the two states (i.e., linear or non-linear).

The second part of the signal (i.e., $\cos\left(\frac{\mathrm{d}f}{\mathrm{d}t}(t)\,t\right)$) introduces a vibrotactile component (i.e., small oscillations) to improve the level of discrimination between the two states. Indeed, previous studies showed the efficiency of the vibrotactile feedback to display the evolution of local features [7].

Based on this haptic stimulus, we proposed to carry out an experimental study to evaluate the contribution of the vibrotactile feedback for the improvement of discrimination between linear and non-linear physical based signals.

2 Experimental study

Based on the proposed haptic stimulus and objectives defined above, we investigated the following hypothesis.
-**H1**: subjects discriminate more effectively the two states with the vibrotactile feedback.
-**H2**: subjects discriminate faster the two states with the vibrotactile feedback.

2.1 Hardware and Software setup

The experimental platform is based on a standard desktop station. The haptic device, a Sensable PHANToM Omni, is placed on the desk. The software part includes two components. First, the graphic interface to control the progress of the experiment (i.e., activation of the haptic signal display), and to get answers from participants for each presented stimulus (that is nature of the divergence: linear or non-linear). Second, the haptic module supports the display of the haptic stimulus according to the investigated conditions (see below).

2.2 Participants

Eleven participants (nine men and two women) recruited at the LIMSI lab (University of Paris-Sud), aged between 24 and 57 years old, completed the experiment.

2.3 Experimental conditions

The main evaluated variable concerns the presence of the vibrotactile mode feedback (i.e., with or without the vibrotactile feedback). The kinaesthetic feedback is always present. This variable presents the following conditions (see figure 2):
-C1: Haptic feedback without the vibrotactile mode
-C2: Haptic feedback with the vibrotactile mode

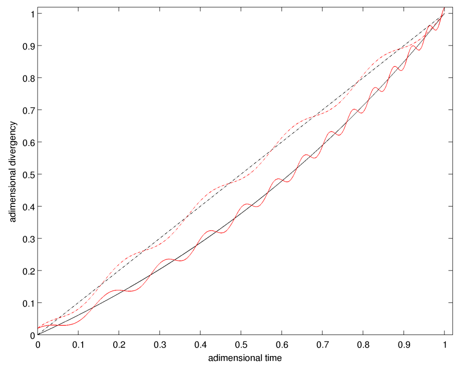


Fig. 2. Black and red curves correspond to the haptic stimulus without and with the vibrotactile mode respectively. Dash-dot curves refer to the linear signal (quasi-periodic state, herefore red curve has a constant frequency), while curves with solid lines correspond to the non-linear signal (chaos state, herefore red curve has an exponential increase of its frequency).

Measures The following measures were collected for each presented stimulus:
-M1: Perceived signal – Participants must select a single answer (linear signal or non-linear signal)
-M2: Completion time – taken time to give the answer

In addition to these measures, we present to participants a short questionnaire for the subjective evaluation of the proposed stimuli, investigating participants' confidence in their response.

Procedure Subjects sit in the front of the desk and hold the haptic arm with the dominant hand. They interact with the graphic interface with the non-dominant hand (keyboard) 2.3.

Before the experiment, we describe the theoretical context, the presented haptic feedbacks, and the way to answer with the graphic interface. The experiment begins with a training case. Then, forty stimuli, with random parameters uniformly distributed – on ranges such as the distinction between linear signals and exponential's ones is non-trivial – are presented to subjects. For each case, they activate the haptic perception, then they give the answer through the graphic interface. The experiment ends with a short questionnaire. The presented haptic stimulus is along the horizontal axis to improve the perception of vibrations [8].

Fig. 3. Experimental setup: the user holds the haptic arm with the dominant hand and interacts with the graphic interface with the non-dominant hand (keyboard). The box prevents the visual observation of haptic stimuli (i.e., movements and oscillations)

3 Results

As the population does not respect a normal distribution, we used a non-parametric statistical test: Wilcoxon signed-rank test.

Completion time Taking into account all answers (i.e., right and wrong answers), the results showed a highly significant reduction of the response time of 20% from **C1** to **C2** (9.9 s compared to 8.2 s, $p-value \ll 0.01$). If we take into account only right answers (i.e. correct identification of the signal), the vibrotactile mode **C2** reduced significantly the completion time of 12% (from 8.8 s for **C1** to 7.9 s for **C2**, $p-value = 0.07$). These results showed that the vibrotactile mode improves the reactivity of users, and enables a more rapid understanding of the signal nature. This confirms hypothesis **H2**.

Discrimination efficiency The results showed an overall and highly significant improvement for the identification of the right signal of 21% from **C1** to **C2** (0.67 compared to 0.8 right identifications, $p - value \ll 0.01$). This confirms hypothesis **H1**. Advanced statistical analysis showed that the required time for correct identifications is lower than for wrong identifications. Subjects are 25% faster under the **C2** condition ($p - value < 0.05$), and 36% faster under the **C1** condition ($p - value \ll 0.01$). In fact, the answer time cannot be reduced too drastically, as the mathematical difference between signals collapsed to zero. Then, there is a natural time contraction as **H2** is verified, explaining why the time reduction under **C1** is larger than the time reduction under **C2**.

Finally, the subjective evaluation showed that subjects are more confident for their answers with the vibrotactile mode (from 1.6 for **C1** to 2.8 for **C2**, interval from 0– not confident at all– to 4 – very confident–, $p - value < 0.05$).

4 Conclusion

This paper presented a suitable combination of kinaesthetic and vibrotactile stimuli to improve the level of discrimination between linear and no-linear functions. These functions, corresponding to the local divergency rate, provide relevant and complementary information to understand the features of explored attractors. The results showed a significant improvement of the level of identification for each signal. Moreover, users understand and perceive more rapidly the nature of signal. Based on these results, the next works will, on the one hand, explore the differential threshold for the distinguishment between the presented signals. One the second hand, we will investigate the combination of the visual feedbacks (i.e., 3D representations of attractors) and haptic stimuli for the exploration of physical state space presenting several physical relevent properties, such as attractors, Lyapunov–based coherent structures (Lagrangian Structures) etc.

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